A Typology of Undergraduate Students' Conceptions of Size and Scale: Identifying and Characterizing Conceptual Variation

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Abstract: The importance of ‘‘size and scale’’ in nanoscience and engineering has been recognized by both scientists and science educators. A solid understanding of this concept is key to the learning of nanoscience. Students, however, have been reported to have considerable difficulty grasping this concept; yet little is known regarding their state of understanding. To address this knowledge gap, we conducted a series of studies that were aimed at exploring the different ways students conceive of ‘‘size and scale’’ in the context of undergraduate nanoscience and engineering courses. Informed by Variation Theory of Learning (Marton and Booth, 1997), we identified four major categories (with two sub-categories within each) of student conception—fragmented, linear, proportional, and logarithmic. These conception categories, together with the aspects of variation that characterize and distinguish them, are summarized in a typology. In addition to serving as a diagnostic tool to describe students' understanding, this typology can also be used to guide the development of instructional interventions that facilitate students to move toward a more sophisticated understanding of ‘‘size and scale.’’ © 2010 Wiley Periodicals, Inc. J Res Sci Teach 48: 512–533, 2011
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The field of nanoscience deals with objects and structures in the range of 1–100 nm, a size range that is rarely (if at all) considered in everyday practice. Thus, whether and how students come to grasp phenomena of such small *size*—defined as ‘‘the extent, bulk or amount of something’’ (Stevens, Sutherland, & Krajcik, 2009)—is a critical issue in nanoscience education. Closely associated with the idea of size is the concept of *scale*, which refers to the conventionally defined numerical representational systems of size such as the linear scale and the logarithmic scale. Due to the invisible nature of objects at the nanoscale, people need to rely on numerical scales to conceptualize how small they are. Therefore, while size and scale are two separate concepts in many situations, they are treated as one unifying idea in this paper due to the specific context of nanoscience.

‘‘Size and scale’’ has been recognized as a fundamental theme in various science education standards such as the American Association for the Advancement of Science (AAAS) Project 2061 and the *Benchmarks for Science Literacy* (AAAS, 1993). A solid understanding of this concept is reported to play a critical role in their work by professionals from a wide range of fields (Jones & Taylor, 2009). Recent national workshops with nanoscientists and education experts also identified ‘‘size and scale’’ as one of the ‘‘big ideas’’ both at K-12 and undergraduate levels (Stevens et al., 2009; Wansom et al., 2009). A sophisticated understanding of ‘‘size and scale’’ not only helps students better grasp the size range in which nanoscience operates, but facilitates their learning of size-dependent properties that are arguably at the core of nanoscience. Without a clear understanding of ‘‘size and scale,’’ it is difficult for students to understand the mechanisms behind why objects or materials behave differently at the nanoscale.

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Theoretical Framework

Previous Research on "Size and Scale"

Though relatively few studies have examined students' understanding of "size and scale" in the science context, a consistent finding to date is that people have difficulty comprehending and comparing sizes, particularly extremely small or large ones (Delgado, Stevens, Shin, Yunker, & Krajcik, 2007; Tretter, Jones, Andre, Negishi, & Minogue, 2006). Tretter and colleagues (2006) provided students (elementary to graduate level) with objects from the atomic scale to the interplanetary scale, and asked them to assess the size range (e.g., 1–10 m) for each object. Interviews conducted with students while they completed the task of grouping objects of similar sizes together revealed that students of all ages assigned relative rankings of objects more accurately than absolute rankings. Conceptual boundaries or "size landmarks" separating distinctly different size categories were also identified for all age groups. Boundaries at the small end of the size spectrum appeared to be a source of confusion and this difficulty was more pronounced for younger students who were unable to rely on measurement units (e.g., mm, nm) to conceptualize different size categories. Similar findings have been reported by other researchers such as Castellini and colleagues (2007). A larger scale survey ($n = 1{,}500$) with similar questions (Waldron, Spencer, & Batt, 2006) also reported that adults and children had difficulty ranking small, invisible objects (i.e., germ, molecule), even though they had more success ordering the measurement units (i.e., micrometer, nanometer) corresponding to these objects. Conceptual difficulty with the small end of the scale was found in a study of middle school students (Jones et al., 2007). Possible reasons reported for this difficulty included the invisible nature of small objects, the negative number in the exponential notation, and the unfamiliar names of measurements for these objects. It seems, as the authors stated, that "there is a point where small is just *undifferentiated small* to students" (p.192).

Researchers have also explored how people make sense of size differences in quantitative terms. Delgado and colleagues (2007) investigated how middle and high school students conceptualized objects' size differences in terms of how many times bigger/smaller. Participants sorted cards (representing objects of varying sizes), estimated how many times bigger/smaller each object was compared to a reference object (i.e., pin head), and then assigned absolute sizes to each object. "Nearly two-third of the students interviewed did not perceive a logical, necessary connection between the sizes of two objects and the number of times bigger one object is than another" (p.26). Less than 5% of the participants were able to generate answers regarding absolute sizes consistent with the number of times bigger/smaller they reported the object to be relative to the pinhead. Tretter, Jones, and Minogue (2006) asked students ranging from elementary to graduate level to name objects representing a wide range of sizes in increments of factors of 10 near the human body size range, or factors of 1,000 further away from this range. Students performed well in the range near human body size, but poorly as size increased or decreased. Accuracy was particularly low in the micron range. Participants tended to name objects that were too small at the very large end and objects that were too large at the very small end of the size spectrum. Similar results were also found in a related study with preservice and experienced science teachers (Jones, Tretter, Taylor, & Oppewal, 2008).

The importance of proportional reasoning ability in understanding "size and scale" has been explored in recent studies. In a study that examined the impact of the film *Powers of Ten* (Eames Office, 2009) on middle school students' understanding of "size and scale" (Jones et al., 2007), students' proportional reasoning ability was found to be positively correlated with their accuracy of ordering objects and assigning them with correct size labels. Taylor's dissertation study (2008) also found that performance on proportional reasoning assessment was significantly correlated with the ability to understand the concept "surface area to volume ratio," which is closely related to "size and scale".

Variation Theory of Learning

Our study sought to describe not only how students understand the core idea of "size" and "scale," but more importantly, how they comprehend the attributes associated with them and the relation between the attributes. In order to do so, we employed Variation Theory of Learning (Marton & Booth, 1997; Marton & Pang, 2006; Marton & Tsui, 2004; Pang & Marton, 2005) to guide our research, particularly in terms of data analysis and interpretation.

According to Variation Theory, conceptions are viewed in terms of awareness, and learning in terms of changes in the learner's awareness structure as it relates to a concept or a phenomenon. That is, different conceptions correspond to awareness at different levels of complexity derived from different ways of experiencing a phenomenon. Variation between conceptions is due to the different aspects of the phenomenon that the learner is able to discern. Variation in learners' awareness or learners' experience of a phenomenon can be described in terms of *Aspects of Variation*. Research following a Variation Theory approach leads to a hierarchical set of increasingly more complex categories of conception, complemented by a set of aspects of variation that highlight the critical distinctions between the conceptions. The more sophisticated the conception is, the more awareness of aspects of variation it demonstrates. For example, Pang and Marton (2005) explored students' understanding of price fluctuation as affected by demand and supply. Five conceptions increasing in level of sophistication were identified—consideration of change in features of the goods only, consideration of change in demand only, consideration of change in supply only, consideration of change in both demand and supply but not their magnitudes of change, and consideration of change in both demand and supply as well as the relative magnitudes of change. The corresponding critical aspects of variation were change in demand/supply (directionality), magnitudes of changes in demand/supply, and the magnitudes of change in relative terms. The least sophisticated conception exhibited awareness of none of these aspects of variation, and the most sophisticated one demonstrated full awareness.

In such an approach, all conceptions bear important information to describe and understand the "outcome space." Therefore, alternative conceptions are not treated as flawed and needing to be replaced by the scientific models or theories (Smith, diSessa, & Roschelle, 1993), but valuable knowledge elements that serve as foundation for students to develop and expand their knowledge system (Vosniadou & Verschaffel, 2004).

Study Goal

In this paper, we report a series of three studies that explored how undergraduate students made sense of the concept "size and scale," particularly as it relates to the context of nanoscience. We sought to understand not only how students ordered and grouped objects based on their size, but more importantly, how they conceptualized size differences, and represented them with numerical scales. Such knowledge can contribute to the design of instructional interventions that promote appropriate use of numerical scale, a crucial skill required in many STEM courses and professions.

Study Overview

Three related studies (two interview studies and one survey study) were undertaken to explore undergraduate students' conceptions of "size and scale." It should be emphasized that the studies were exploratory in nature, focusing on relatively small samples of students. Following Variation Theory, each individual student's response was treated as representing a potential valid variation of the participant population, an assumption that is necessary to construct the conceptual space of these students but may be vulnerable given the small sample size.

The initial interview study (Interview study 1) involved a small group of students, and took a qualitative, exploratory approach. The findings led to a more focused follow-up study (Interview study 2), with a larger group of students to further probe their understanding. The second interview study confirmed and extended findings from Interview study 1, and a typology of student conceptions of "size and scale" was developed based on the collective results of the two studies. It should be pointed out that while the interviews in these two studies were conducted in a similar manner, the tasks accompanying the interviews were different, with Interview study 1 requiring students to generate scales themselves and Interview study 2 asking students to choose from a set of given options. Subsequently, a set of multiple-choice assessment items was developed, and administered to a separate larger group of students as a way to test the typology. This step was done in the Survey study, the results of which provided further confirmation of the typology.

Methods

Participants and Procedures

Participants were subsets of students who enrolled in three engineering courses at a major Midwest university—a freshman engineering design course for majors (EDC), an introductory materials science courses for non-majors (MSC-Gen), and an advanced materials science courses for majors (MSC-Adv). All courses had a nanoscience focus, either by including a nanoscience unit or having nanoscience as the unifying theme of the course.

In order to capture the wide variations within students' conceptions, we were particularly concerned with selecting participants that had the potential of exhibiting varied levels of understanding. In the initial interview study, a self-developed ''size and scale'' inventory (see Light et al., 2007 for details) was first administered to EDC and MSC-Gen students, and within each class, students' performance placed them into one of the ''average,'' ''above-average,'' and ''below-average'' groups. Students from each performance group were invited to participate, and the final participants included 12 students (six EDC and six MSC-Gen) that varied in terms of gender make-up and ''size and scale'' understanding. Each student participated in a task-based think-aloud interview, constructed based on tasks used in previous research (Tretter, Jones, Andre, et al., 2006). The participants were asked to order a list of objects of widely varying sizes (the length of a football field, the height of an elephant, the length of a typical science textbook, the width of a human hair, the diameter of a bacterium, the diameter of a virus, and the diameter of a hydrogen atom) along a line, and then to apply a numerical scale to the line to represent their size differences. Students were prompted to verbalize their reasoning as they completed the tasks.

Interview study 2 was aimed to further probe the preliminary findings yielded in Interview study 1. Twenty students—eight from EDC, eight from MSC-Gen, and four from MSC-Adv—participated in this study. As in interview study 1, these students were chosen following the strategy of ''maximum variation sampling'' (Patton, 2001) so that we could include a wide range of students who were likely to hold very different conceptions of ''size and scale,'' and thus increase the chance of capturing all possible variations in their understanding. In this round of interviews, students were provided with four scale options for a similar list of objects as given in interview study 1 (Figure 1), with one of the options allowing students to create their own scale. These scale options were idealized versions of the major conceptions we observed in the Interview study 1, with option A representing a linear-based scale, option B demonstrating a fragmented view of scale (i.e., objects belonging to different ''worlds'' cannot be represented on a continuous scale), and option C indicating a logarithmic-based scale. Students were asked to choose the option that was most appropriate and explain why. All interviews were tape recorded and transcribed verbatim.

Item: A group of students were asked to create a scale that best represents the relative size differences of the following objects: the length of football field (about 91 m), the height of an elephant (about 3 m), a diameter of a human hair (about 0.0001 m), the diameter of a virus (about 0.0000004 m), and the diameter of an atom (about 0.01 m). Here are some examples of what they created. Which one of the following do you think is the most appropriate scale?

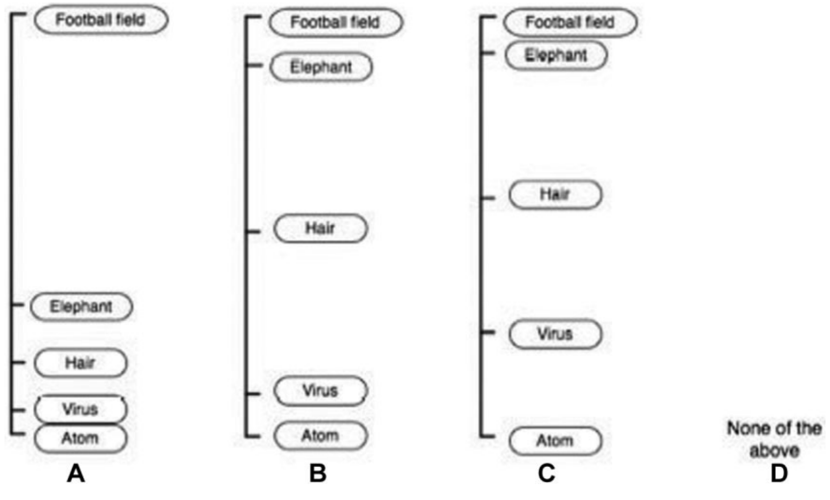

Figure 1. Interview task used in interview study 2.

The Survey study consisted of 111 students—29 from EDC, 70 from MSC-Gen, and 12 from MSC-Adv. Most of the students (over 90%) enrolled in the courses agreed to complete the survey, which explains the variation between number of participants from each course. Three ‘‘size and scale’’ assessment items (Appendix A) were administered to the participants as part of a larger survey. These assessment items were designed to reflect the key aspects of variation identified in the interview studies. Item 1 was essentially the same as the task used in Interview study 2, as it was found to be useful in the interview studies to elicit students’ scale conception(s); items 2 and 3 aimed to differentiate the fragmented versus the continuous conception from two different angles—item 2 looked at whether students who held the fragmented view confused different ‘‘scales’’ with different ‘‘appropriate units of measurement,’’ whereas item 3 examined whether the visibility and properties of objects of widely varying sizes interfered with students’ belief regarding the feasibility of representing them on the same scale. Each question consisted of two parts—a multiple-choice part and a written justification part.

Data Analysis

Given the exploratory nature of the study, the analysis of the Interview study 1 data took a phenomenographical approach (Akerlind, 2005; Bowden & Marton, 1998; Marton & Booth, 1997; Micari, Light, Calkins, & Streitwieser, 2007), which aims at mapping out the qualitatively different ways in which aspects of the phenomenon are experienced, as well as the structural relationships that connect these different ways of experiencing. The coding process included several steps. First, two researchers individually reviewed all interview transcripts and accompanying written artifacts to identify different approaches students took in responding to the interview task (i.e., ordering objects and assigning scale). The variations observed by individual researchers were then compared and discussed before generating an initial set of codes. The initial codes were applied to all transcripts by the same researchers, with the goal of grouping excerpts demonstrating the same ordering and scaling approaches. The researchers then met to compare their coding results, and discussed the similarities within and differences between these groups. As inter-rater discrepancies were few (<10%), the researchers made minor revisions to the codes, and proceeded to apply the revised codes to all the transcript excerpts (additional information of the codes is available as Supplementary Material accompanying the online article). Instead of conducting this round of coding separately, the researchers coded the transcripts together, and resolved any discrepancies through discussions.

Since Interview study 2 was built upon Interview study 1, and less exploratory in nature, the analysis process was thus more focused. Specifically, the initial codes were derived from the observed variations in Interview study 1. One of the two researchers from the previous study applied the initial codes to all transcripts, and excerpts or cases that were not covered by the initial codes were noted. These newly emerged variations were discussed among the two researchers from Interview study 1, and new codes were created to describe them. The final codes were then used to code the transcripts by the same researcher who conducted the initial round of coding (additional information of the codes is available as Supplementary Material accompanying the online article).

For the survey results, we did not treat them quantitatively by simply calculating the number of items in which students chose the most complex answer, as we believe that a particular multiple choice does not, in itself, necessarily warrant the corresponding conceptual understanding it is meant to represent. Instead, students’ multiple-choice and written justifications were reviewed holistically. For item 1, we coded the responses using the same codes used in Interview study 2 (i.e., to characterize conceptions based on the variations observed in the interview studies). For items 2 and 3, initial codes were generated to capture the variation revealed in students’ responses. These codes were applied to a sample of 59 responses (about 50% of the total responses) by two researchers in three rounds. In each round, about one-third of the responses were coded by two researchers. Coding results were compared, discrepancies were discussed, and appropriate revisions were made to the codes (additional information of the codes is available as Supplementary Material accompanying the online article). The final percent agreement between the researchers was 82.6%. Any responses that were irrelevant or insensible were coded as such, and excluded from further analysis.

Results

Interview data suggested four distinctive categories of student conceptions of ‘‘size and scale’’—fragmented, linear, proportional, and logarithmic. Given its larger sample size, we initially expected the survey data to reveal more variations that were not captured in the interviews. However, the brief nature of students’ survey responses often times made classification beyond the aforementioned four categories difficult. As a result, while the Survey study data confirmed the four categories and some of the within-category variations, no new conceptions were revealed. In what follows, we will describe the conceptions in detail, and illustrate them with example interview excerpts and if available, survey responses. Please note that of the three survey items, item 1 was most informative with respect to student conceptions, so the survey responses shown here are primarily from item 1. Items 2 and 3 were more helpful in providing insights on possible reasons for the fragmented conceptions, and thus were only referred to in the discussion of the fragmented category.

Fragmented Conceptions

What separates conceptions within this category from the other categories of conception is whether students understood scale as continuous, or in other words, as capable of representing and ordering objects of widely varying size in a continuous way. This was a rather surprising finding, as we assumed undergraduate students would have established a continuous model of ordering, measuring, and comparing objects. However, the data revealed that while some students held the continuous view, others perceived objects as residing in separate worlds according to their size—namely, the macro, micro, and nano world. Correspondingly, their view of the scale was fragmented—each ‘‘world’’ demands its own distinct scale, and these scales cannot be compared or connected in any way.

In the interviews, the ‘‘separate worlds’’ view mainly manifested itself as the belief that the macro and sub-macro worlds are disconnected or fragmented. This view is illustrated by Katrina’s drawing and comments in response to the interview task (Figure 2). Specifically, she placed macro and sub-macro objects at each end of the scale, with objects in each group relatively close together. And these two groups are separated by a huge ‘‘gap,’’ in this case between textbook and hair. Katrina explained her scale as follows:

> Relatively. . .these (*pointing at the hair, bacterium, virus, and atom*) must be down here at the bottom ‘cause these are so small; these would be all chunked together; and these (*pointing at the football field, elephant, and textbook*) would be more like this. . .but this (*pointing at the gap between hair and textbook*) is not enough space in between.’

This explanation, coupled with the drawing, suggested that in Katrina’s mind, the size differences among macro or sub-macro objects are negligible and can be ‘‘chunked together,’’ but the difference between the two ‘‘worlds’’ is so enormous that they could not be compared using the same scale. Indeed, the two worlds are represented in her drawing by different units of measurement, yards, and feet primarily for the macro world and nanometers for the sub-macro world.

In contrast to Katrina, who included numerical measurement of size to aid her scale construction, some students who held the fragmented view did not even integrate numbers within their graphical representation of a scale. That is, they were only able to generate a linear scale based on a qualitative understanding of objects’ size differences (i.e., football field is a lot bigger than elephant versus a virus is only a tiny bit bigger than an atom), and did not use any numbers (e.g., quantification of size differences on the scale) in their representation of them. As an example, Tom constructed his scale using three pieces of paper taped together, and used the object ‘‘hair’’ as the ‘‘cut-off point’’ or borderline in dividing the ‘‘big worlds’’ and the ‘‘small worlds’’ (Figure 3). He was, however, reluctant to use any numerical system to describe the actual sizes or size differences of the objects even within each of the disconnected worlds:

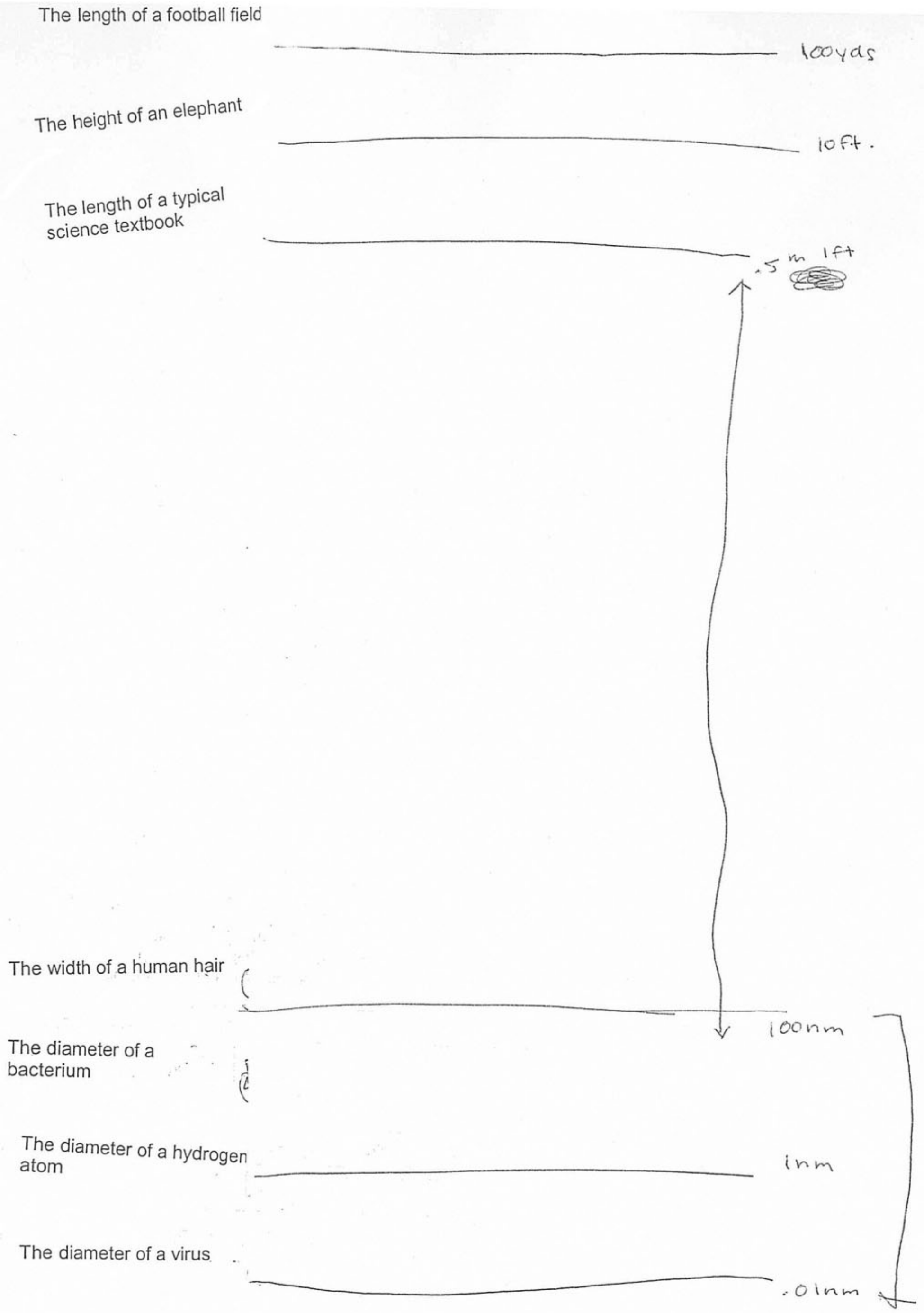

Figure 2. Katrina's scale diagram [The text on the left reads (from top to bottom): the length of a football field; the height of an elephant; the length of a typical science textbook; the width of a human hair; the diameter of a bacterium; the diameter of a hydrogen atom; the diameter of a virus. The text on the right reads (from to bottom): 100 yards; 10 ft; .5 m, 1 ft; 100 nm; 1 nm; 0.01 nm].

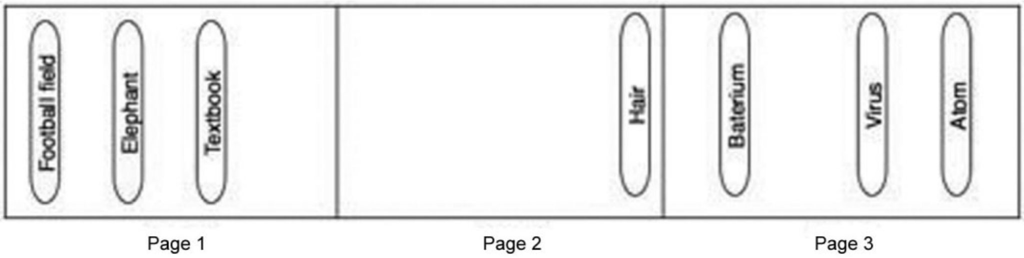

Figure 3. Tom's scale diagram (reproduced).

> Interviewer: Ok, if you were to rearrange these cards a little bit to actually use the distance between these objects to show their relative differences, how would you change this?
>
> Tom: These (*pointing at the smaller end of objects*) would be like way over here, and there would be a sizable amount of difference between them (*pointing at the smaller and the bigger objects*). And these (*pointing at the bigger objects*) would be like way over here on the other side because they are such different in size. Even textbook and human hair, it's like a huge difference between that, more than say between the textbook and the elephant.

Similar dichotomous views regarding the continuous nature of scale were also observed among the survey responses. For survey item 1, 10 out of 79 valid responses suggested a fragmented view. These responses tended to group objects by their visibility or by the ''worlds'' they belong to, as illustrated by Alice's written responses on the survey:

> A football field and an elephant are pretty close on this scale because they're both big (bigger than a human). A hair is somewhere in the middle because it's neither very big or very small (it's still visible to the naked eye). A virus is very small form of life. It's no longer visible to the naked eye.

This explanation, coupled with her choice of option B, suggested that for Alice, the objects were separated into different groups based on visibility, and the size differences between these groups were so dramatic that they could not be represented by a single scale.

Responses to item 2 and 3 suggested possible reasons for the persistence of the fragmented view. One of them, revealed by item 2, is the confusion of ''units'' with ''scale,'' which was seen in 25% of the responses that indicated the view that different units are needed at each end of the scale. That is, these students appeared to equate the appropriate measurement units for objects with different numerical scales to represent them. They believed that if the objects needed to be measured in different units, then they have to be represented using different scales. For instance, Anna chose D (''yard'' at one end and ''nanometer'' at the other), and wrote:

> Yard > Foot > Millimeter > Nanometer; Since the difference between the biggest and the smallest measuring unit is so big (yard compared to nanometer), they cannot both be represented on a single dimension scale. We need a scale that covers both and everything between them.

Another possible reason for the fragmented conception is due to the confusion over the meaning of ''scale,'' namely confusing ''scale'' in ''numerical scale'' with that used in ''macroscale'' or ''nanoscale'' even though they refer to related but different ideas. Alex's response is representative of this confusion:

> Microscale are generally used for objects that humans can see with naked eye. Whereas the nanoscale must be used for objects that humans can't see with their naked eye.

Linear Conceptions

The defining feature of conceptions in this category is that the placement of objects on the scale directly corresponds with one's physical experience or visual observation of the objects' sizes. In the second interview study, Amy, for example, described a view that scale should serve almost as a literal replication of the absolute size differences of the objects, that is, in order to be able to visualize the differences between the tiny objects, a piece of paper is far from sufficient to represent the huge differences between the bigger objects:

> Amy: . . .So the football field is off the charts, first of all, it probably should go to the end of the table. The elephant is also off the charts because these are so much bigger than these (*pointing at the small objects*). For me to show any differentiation between these ones down here (*the small objects*), you have to realize these ones (*the big objects*) are going—maybe not even to the end of the table, maybe to the bookshelves, or maybe into the next room, but. . .
>
> Interviewer: You mean football field. . .
>
> Amy: . . . for the football field. So that one goes way out. This one (*elephant*) also does, too.
>
> Interviewer: Like the bookshelf?
>
> Amy: Yeah, I mean, the football field is going to go the farthest away. The elephant, maybe the elephant goes to the end of this table, and the football field goes to the bookshelf. The textbook, maybe to where your recorder is. Hair here *(pointing at a location slightly closer to her than the recorder).* The virus, I think, is smaller, and then this one *(atom)* is like that *(pointing at a location much closer to her).*

In some cases, the choice of a linear scale was not as clear as in Amy's case. Rather, it was ''masked'' by features that seemed to indicate a more advanced scale choice. Katie's interview response from Interview study 1 provided a good example—she produced a drawing that resembled a logarithmic scale (Figure 4), and she introduced it as such. Even when initially probed, as we can see in the excerpt below, she seemed to be able to rationalize why the logarithmic scale was the appropriate choice. However, when further probed, Katie's explanation suggested that her conception was actually guided by a linear sense of scale instead.

> Interviewer: So, before you said that this is on the logarithmic scale, why did you choose the logarithmic scale here?
>
> Katie: The diameter of a hydrogen atom, in comparison with the length of the football field, is like. . .you can't use the same scale to compare them, because the hydrogen atom is so much smaller. . .
>
> Interviewer: Ok. Could you explain it a little bit, like I see between some objects the space is smaller than others? Could you explain it?
>
> Katie: I think it's just might be my drawing of those, but I guess if I were to do it over again, these (*referring to the sub-macro objects*) would be much closer together, and as it gets larger, they (*referring to the macro objects*) would spread out.
>
> Interviewer: Why would that be?
>
> Katie: Because 1 m is much bigger than 1 nm.

In this case, even though Katie claimed her scale to be logarithmic and used log-like notations for her scale, the statement ''because 1 m is much bigger than 1 nm'' revealed that she was still operating under the belief that smaller size difference corresponds to shorter distance on the scale whereas bigger size difference corresponds to longer one, a clear indication of the linear conception. This example raises the issue that the employment of log terminology or notation is not necessarily indicative of a logarithmic conception, and the underlying conception could be quite different, as illustrated by Katie's example here and Karen's example later in this section.

Item 1 of the survey provided additional support that the linear conception indeed represents many students' (25 out of 79 valid responses) understanding. The majority of these students ($n = 22$) indicated a linear conception without references to the log-related terminology or components. For example, Mike's drawing (Figure 5) suggested that on his ideal scale, pairs of objects with the larger absolute size difference should have bigger gaps between them, whereas pairs with smaller size difference should have correspondingly smaller gaps—a clear indication of the linear conception.

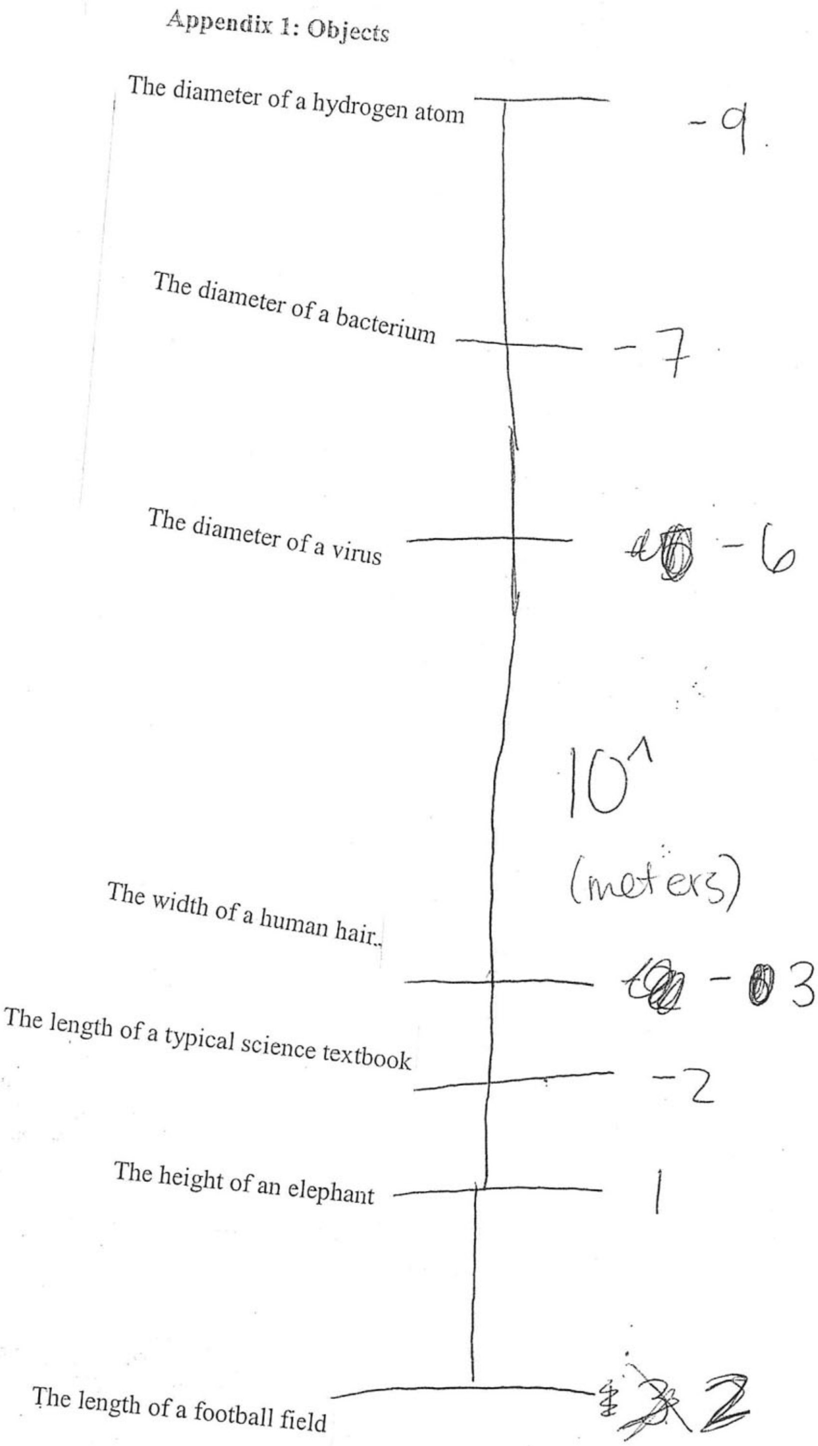

Figure 4. Katie's scale diagram.

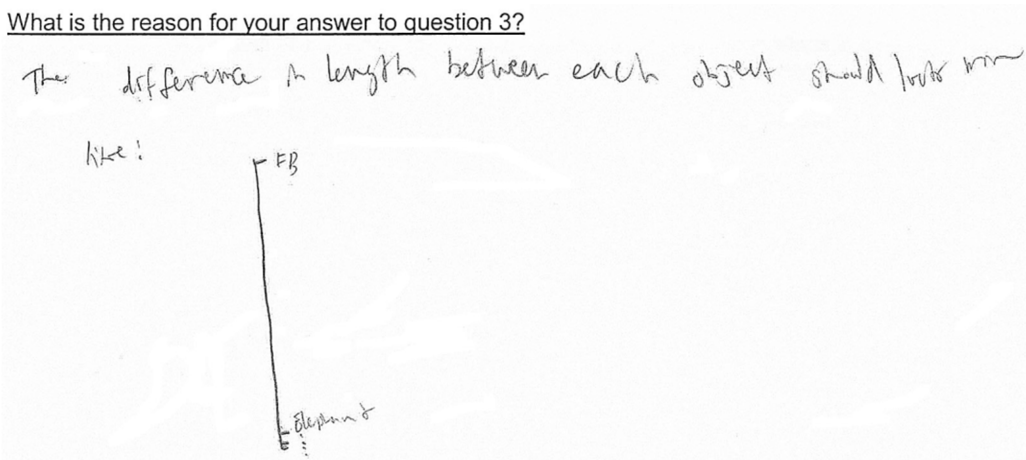

Figure 5. Mike's survey drawing (The text reads: ''The difference in length between each object should look more like:'').

The remaining three survey responses were like Katie's in that the students chose the log scale for the multiple choice part or used related terminology in describing their choices, but their thinking was essentially still based on a linear understanding of scale. In contrast to Mike who exhibited straightforward linear conceptions, these conceptions were characterized by the use of ''powers of 10'' or logarithmic notions that were grafted onto the linear conception. For example, Yan wrote in response to Item 1:

> The scale has to represent the position of the object of a scale that is to represent the power of ten. The spacings in A is fine to make that representation.

Even though he used the terms ''Powers of Ten,'' his choice of A, a representation of the linear scale, revealed that he did not understand how the spacing in a log system worked and was not truly comparing the objects using such a scale or any scale system based on proportions, but rather was grafting terminology from such system onto a linear understanding.

Proportional Conceptions

As mentioned earlier, we observed in the interviews scale descriptions and representations that seemed to combine features of the linear scale and the logarithmic scale. Unlike Katie, whose ''hybrid'' scale turned out to be constructed based on the linear conception, other students built their ''hybrid'' scales based on the *proportional differences* of objects' sizes (i.e., how many times bigger or smaller one object is in comparison to the others), as opposed to the absolute differences. These scales are of particular interest because they may suggest a path of progression from the linear scale to its logarithmic counterpart. We provide two examples here to demonstrate this type of conceptions, and furthermore, to illustrate the subtle differences in terms of how students incorporated the idea of ''proportion'' in their scale construction.

Jeremy in Interview study 2 described his choice of scale using the idea of proportion. That is, his construction of the scale was based on *how many times*, as opposed to *how many meters (or other units of choice)*, bigger or smaller an object was in comparison to others. Jeremy also provided valid reasons for why the use of proportion or in his word ''times'' was advantageous for comparing objects of widely varying sizes:

Jeremy: But when we're going times, it's probably—like (for) one elephant to reach to a football field, it probably takes a while. . .I don't know, 25 elephants, 25 times. A textbook is probably about maybe a thousand times. But this stuff *(the small objects)* is like 10,000, 100,000 (times). . .

(*When asked why using 'times' in comparison)*

Jeremy: Yeah. Because you could say a virus is not a centimeter; an atom is not a centimeter. But we both know they're not a centimeter. And, like I said, a virus is not a millimeter. An atom's not a millimeter, but we both know they're small. But what's the difference from a virus and an atom? Only times would show us the difference, how small. If we don't do times, nothing will show us how. We couldn't think mathematically how small an atom is.

The use of proportion can be regarded as more advanced than the previously described linear scale, including the example of a linear scale "dressed up" with log terminology, because this conception recognizes the idea of relative difference (as opposed to absolute difference) that is much more appropriate in comparing and representing objects of widely varying size. Yet students holding this type of conception are missing or appear to be unaware of one key aspect of the log scale—a common factor (i.e., 10) to serve as the reference for comparison. When using proportion as the base for the scale, as the proportion between each pair of objects varies, the reference unit for the scale also varies depending on the particular object pair under consideration. Thus, it can only provide a rough, qualitative description of how different the objects are, but not an accurate portrait of their size differences.

Karen's responses in Interview study 1 provide a good example of the use of the common factor of 10. In this case, she produced a scale that mimicked the logarithmic scale (Figure 6), demonstrating a basic understanding of the "powers of 10," the basis for the logarithmic scale. Nevertheless, her representation and responses did not demonstrate a logarithmic conception of scale because while she was able to order and roughly space out the objects according to the value of their "power" (as in "powers of 10"), as opposed to absolute difference of their sizes, she did not seem to understand the construction of the logarithmic scale, particularly the idea of "equal spacing" between each increment of power. When probed, she verbally revised the placement of the objects on her scale reflecting a confusion caused by the strong influence of the linear scale on her thinking:

Karen: It (*referring to the scale*) would be on the magnitudes of 10, for sure. Oh yeah, for sure.

Interviewer: And why do you think it's the best one?

Karen: I mean because conceptualizing it. . .it's hard for example to put any other certain numerical scale to the difference between a hydrogen atom and the length of a football field, and when you write like 10 to the something (*writing while talking*), you have this other number to sort of judge it by, so it's always 10 to the something, 10 to the something. So ok, -4 is got to be smaller than 4, so obviously that's smaller. . .Why would put in that scale? I think you get a huge range, and for a standard, I think it's always going to be like 10. Did that answer it for you?

Interviewer: Ok? Just looking at the way you ordered them, some you've got a little closer. . .There is a little bit of variation to the length of the distance between.

Karen: Oh that's just chance. If I were to make them more clumped together, it would be like these two would be together, the virus and the bacterium. The human hair would be kind of hang out around here (*pointing at the graphs while talking*). The science textbook would way out over here because you've got one hanging out between the two. So this (*referring to the hair*) is below 1, and that's (*referring to the textbook*) above 1, so obviously makes a huge difference. The height of an elephant, maybe like that, like that (*pointing at the upper end of the line*). It's more picture, graphically accurate representation.

What ties these two examples together is that the chosen scales were both built on proportion-based thinking. Unlike students who calculated size differences by subtraction and allocated the distance between objects on the scale according to their absolute sizes differences, Jeremy and Karen figured out the *proportional differences* between objects' sizes—either by using direct division or using the "powers of 10" system, a key component of logarithmic scale.

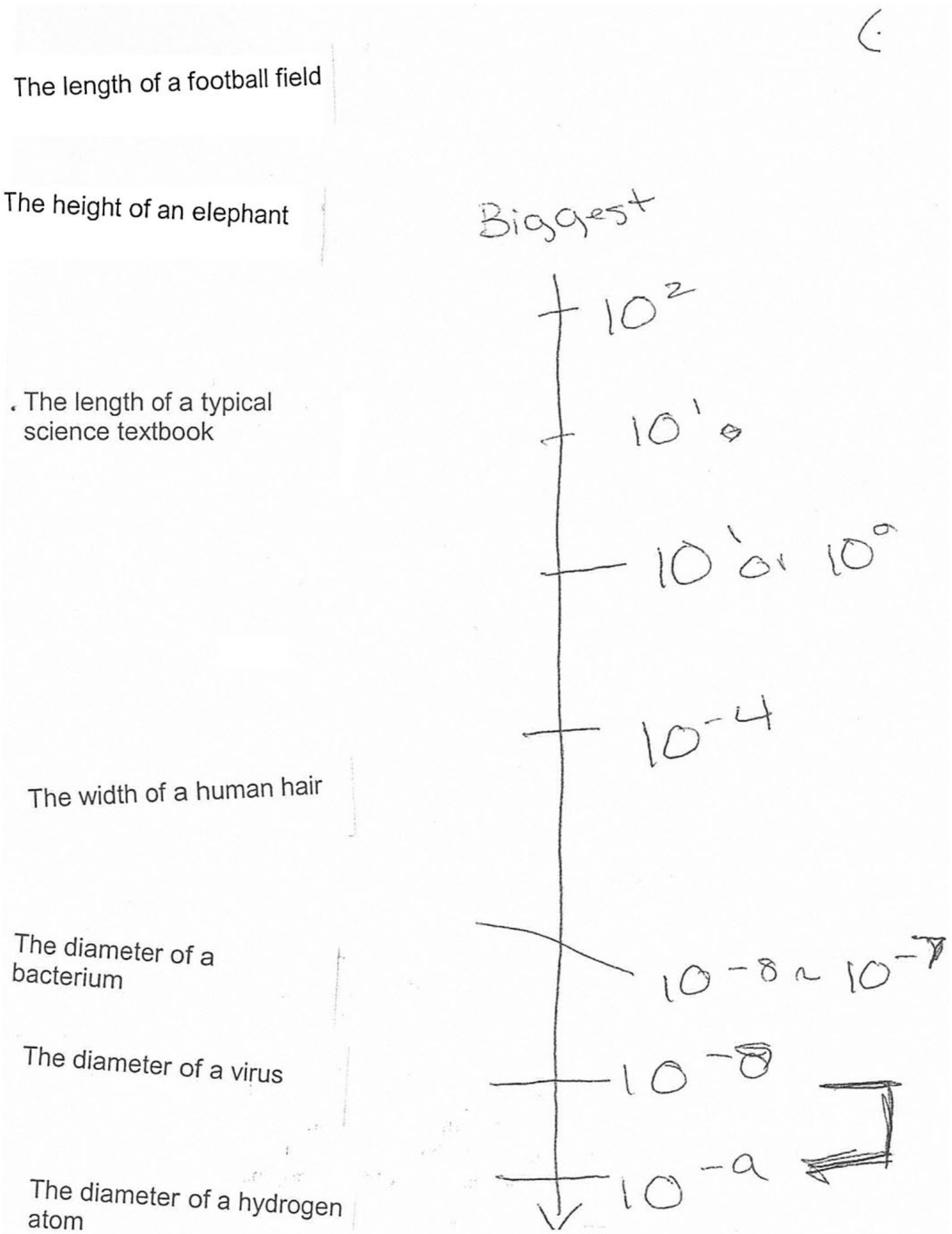

Figure 6. Karen's scale diagram.

Evidence from the survey data also suggested that proportional conception is another major way students conceive of size and scale. Specifically, in response to item 1, 26 students based their scale choice on proportional differences between object sizes, though the comparisons did not involve a common factor. These responses conveyed similar understanding as Jeremy's:

> Sam: It is reasonable to use C, because the scaling is appropriate. For example, football field and elephant should be close, as a football field is 30 times larger, whereas the virus is 4000 times larger than an atom, creating a considerable distance.

Fifteen responses, on the other hand, went beyond the simple calculation of proportions, and employed a common factor of 10 as the reference for comparison. It should be pointed out that since most of these students chose C (the option intended to represent the log scale), and their written responses were brief, we could not be absolutely certain that they did not possess a true log understanding. However, we were cautious about giving these responses such ''benefit of the doubt,'' as Karen's case (Figure 5) demonstrated that someone who understood the idea of ''powers of 10'' could still fail to recognize a crucial feature of the logarithmic scale—the equally spaced intervals that represent the common ''power'' factor (i.e., 10) as in the ''powers of 10'' system. Therefore, we took a more conservative stance, and characterized these conception as still under the idea of proportion, the only difference being that now the calculations were rounded to factors of 10:

> Larry: C is the best answer. Hair is 1000 times smaller than the elephant. Virus is 10,000 times smaller than hair. Atom is 1000 times smaller than virus.

Logarithmic Conception

The most sophisticated conception of size and scale we observed is the logarithmic scale. Unlike the linear scale, the distances between pairs of objects on a logarithmic scale do not mirror their subtractive size differences in reality. The location of objects on this scale is calculated based on the logarithm of the objects' sizes. Kevin's interview quotes below suggested an understanding of how to convert object size into the logarithmic system, and why this is advantageous for objects of dramatically different size differences:

> Kevin: A logarithmic scale is going in steps of multiples of 10, so here you start at 100 m, 10 times smaller would be 10 m, 10 times smaller than that would be 1 m. You just go down by factors of 10 until we get down to the nanometer. And this is what you were saying, like the length of a football field and the height of an elephant maybe isn't. . .the difference between these two is 90 m. . .so the difference between these two (*referring to elephant and textbook*) is 9 m. . .so of course the bigger difference you can't represent that by placing them 90 m apart, then 9 m apart. If you say that they're proportionally different, you can show that using a logarithmic scale. And larger stuff is on top and the smaller stuff is at the bottom. And you could just roughly estimate how large they are.

Nathan's interview responses from Interview study 2 below, in addition, provided a clear indication of how the size differences represented in terms of ''powers of 10'' was translated into appropriate object spacing on the log scale. That is, each distance unit (often shown as ''tick marks'') on the logarithmic scale corresponds to one ''power'' (as in ''powers of 10''), and thus the distance between objects on the scale represents the number of ''powers'' on which they differ.

> Nathan: Because the difference, for example, between a football field and the elephant is only ten times bigger than the difference between an elephant and a human hair. So human hair is like ten to negative 3 m. Then the elephant, think of it as 10 m, so it's like ten to the first, so it *(the ratio between hair and elephant)* is like four times. And this distance (on the scale between the two objects) corresponds to almost one to four. Then the difference between a human hair and a virus, 0.4 μm, so the micrometer is ten to a negative six. . .so, again, the difference in times. . . So the distance between the virus and the elephant is ten to negative seven, and seven is the distance. Like the ratio. . .and the ratio of elephant to the football field is one, ten to the first, so it looks like seven times this distance. Seven times here. And about atom, 10 nm. This should be ten to negative eight. . .

Among students who demonstrated a logarithmic conception, there seems to be an interesting epistemological distinction. Specifically, even though they understood that both the linear scale and the log scale can be used to represent size differences (which reflected a rather sophisticated conception of the scales), only some students seemed to truly understand that both scales were reasonable representational tools, and were aware of the advantage of each scale depending on the objects to be represented or the purpose of representation. For instance, Walter chose both A (the option intended to represent the linear scale) and C (the option intended to represent the log scale) for the Interview study 2 task, and explained as follows:

> Walter: Okay. Let me call these one, two and three. I think whoever did number one (*the linear scale*) did a pretty good job in approaching it from an absolute standpoint. . . .And if I was to draw a football field, I might draw this big. Of course, when you're drawing really small things like atoms or whatever, you have to divide this 1 m, because this is an absolute scale, right? So you have to divide this 1 m into exceptionally small increments. So it's not very effective at modeling the small scale. If I was, on the other hand, to change this to 1 mm, this scale, it's just as valid a scale but, on the other hand, nanometer would have to be one-millionth of this still. And if I wanted to draw an elephant, it would be a lot longer than this. Not a very good idea if you want to draw a scale of many things at once.
>
> Walter: A much more effective one would be powers of ten. And what I mean by that is it's like an exponential scale or a logarithmic scale. So the way that might work is I might draw a line, and then I would say—let's call this ten to the zero—so ten to zero meters. And over here I have ten to the 1 m and ten squared, three, four, five, six, seven. And over here I had a negative one, negative two, negative three, negative four, negative five, negative six. So over here, ten to the zero meters; that's 1 m, but on this scale from ten to the negative seven to ten to the seven, are pretty close in size. And then ten squared, even though it's a football field, which is so much bigger than a human, it is just two notches over. And then ten to the one meter, that might be like a building height, maybe like a two-storey building. Ten to the fifth meters might be like a city block. . . But what I'm trying to say is that this scale you can go very very big, very very easily. And then, on the other side, you can go very very small, very very easily.

It is clear that Walter recognized the possible application of both scales, even though preference was given to the log scale. We deemed such students as having an epistemologically integrated understanding of the linear and the log scale. Walter's example contrasted with Joe's understanding below, who despite demonstrating a logarithmic conception size and scale through accurately drawing and explaining a logarithmic scale, believed it "distorts" the size of the objects:

> Joe: I think a log scale kind of distorts how it looks.
>
> Interviewer: What do you mean by "log scale"?
>
> Joe: A log scale is. . .it goes by factors of ten. So this would be ten to the zero, and then you have ten to the one, ten to the two, ten to the three, and so forth.
>
> Interviewer: So why would that distort the size of the objects, you said?
>
> Joe: Well, if you were to write these out, this is one, this is ten, a hundred. So if you just looked at this, you're saying that the difference between one thousand and ten thousand is the same as the difference between one and ten, and even though this is a lot more.
>
> Interviewer: So why do people use it?
>
> Joe: They use it because it's easier to plot things that go for. . .that have a lot of small pieces down at, say, this area *(smaller end)*, and goes all the way up to this area *(bigger end)*, so that they can plot it on a reasonable size.

It is interesting that for students like Joe, the linear scale was considered as being "real" or "normal," and the log scale was viewed as an "artificial" one created for computational convenience or scientific purposes.

Seven responses (out of 79) to survey item 1 were categorized as demonstrating a logarithmic conception. Unfortunately, due to the brief nature of the survey responses, we were unable to distinguish among these responses whether the understanding of logarithmic and linear scales were integrated. However, we still witnessed some indications in terms of how students viewed and connected the logarithmic and the linear scale. For instance, Kevin, who chose C (the intended log scale), wrote the following on the survey, which hinted at an integrated understanding of the log scale and a scale based on visibility:

> *(The objects could be)* Divided by (1) visible, (2) barely visible, (3) invisible; OR log scale is fine too.

Kevin's response contrasts with Sally's below, which seemed to express the belief that only the linear scale is realistic or normal:

> I used the based 10 scale just because I tend to like to transform things into numbers that are more manageable. Another possible scale could be A, which probably just uses the *normal* scale that most people would probably pick.

A Typology of Student Understanding of "Size and Scale"

It was our goal to explore the different ways in which undergraduate students understand "size and scale" and the structural relationships characterizing and distinguishing those understandings from each other. As shown above, our data revealed four conception categories based on students' descriptions, explanations, representations, and choices of scale for objects of vastly different sizes ranging from an atom to a football field—fragmented, linear, proportional, and logarithmic. In addition, the subtle differences between the examples within the same category suggested the necessity of establishing sub-categories. The typology of conceptions (Figure 7) provides a map of these categories and sub-categories of conception, and describes the key aspects of variation that differentiates between them.

As Figure 7 shows, the main categories of conception (1–4) are listed horizontally in the header row, with two sub-categories (1a-1b, 2a-2b, 3a-3b, 4a-4b) within each category. Within the logarithmic category, the sub-categories 1a and 1b distinguish between 1a student conceptions (e.g., Walter), which demonstrate an awareness of the applicability and epistemological equivalence of both the logarithmic scale and the linear scale (though viewing the former as more advantageous and appropriate in the context), and 1b conceptions (e.g., Joe) which view the logarithmic scale as an invention suited for scientific purposes only as opposed to the linear scale which is more realistic. Students with conceptions in sub-categories 2a and 2b all recognize the importance of scale based on proportional differences of objects' sizes, but they differ in that students with 2a conceptions (e.g., Karen) calculated the proportional differences based on a common factor of 10, whereas students with 2b conceptions (e.g., Jeremy) did the proportional calculation by using simple divisions. Within the linear conception category, sub-category 3a conceptions (e.g., Katie) are distinguished by an awareness of some aspects of logarithmic scale, whereas sub-category 3b conceptions (e.g., Amy) lack such awareness. In the fragmented category of conceptions, sub-category 4a (e.g., Katrina) and 4b (e.g., Tom) distinguish between students who are able to integrate numerical measurements within their scale and those who are unaware of the use of numbers.

The order in which the categories are arranged (i.e., 4b at the left end to 1a at the right end[1]) corresponds to a hierarchical progression of less to more sophisticated understanding. The advance in sophistication is described by the increased complexity of student understanding in terms of their awareness of the key aspects of variation differentiating the categories and sub-categories of conception. There are seven aspects of variation which are listed in the first column of the typology. For example, conception 4b describes the view that scale orders detached "worlds" in a linear fashion, but with no actual numbers associated with such ordering. This conception contrasts with the other categories in terms of the first aspect of variation "integration of number": all the other categories of conception (4a-1a) recognize the integration of numbers with scale (regardless of the type of scale chosen). Similarly, the aspect of variation "proportion" sets apart categories up to 3a from categories 2b and above—the conceptions in latter categories all exhibit the awareness of the role of proportion in the understanding of scale, whereas the former do not. The most advanced conceptions display an awareness of all seven aspect of variation.

While these aspects of variation collectively describe the relationship between the conception categories, we view some of them as playing a more important role in defining the structure of the typology. The aspects of variation "continuum," "proportion," and "equally spaced intervals" are such aspects,

Categories of conception		Fragmented *Scale is viewed as a fragmented structure, unable to connect different "worlds"*		Linear *Scale is understood as having a linear structure representing direct observation or experience*		Proportional *Scale is understood as having a structure representing proportional differences*		Logarithmic *Scale is understood as having a logarithmic structure representing "powers of 10"-based differences*	
Aspects of variation		4b	4a	3b	3a	2b	2a	1b	1a
1	Integration of numbers	Scale orders detached "worlds" w/o numbers	Understands scale as ordering or connecting objects of different "worlds" using numbers						
2	Continuum		Scale orders detached "worlds" w/ numbers	Understands scale as a continuum linking objects of widely varying sizes					
3	Log scale awareness			Linear scale w/o reference to log component(s)	Understands scale as incorporating certain log scale components (e.g. terminology, visual representation, basis of construction)				
4	Proportion				Linear scale w/o indication of proportion-based thinking	Understands scale as based on proportion or ratio			
5	Powers of 10 (P10)					Proportion-based scale w/o common factor	Understands proportion in terms of P10		
6	Equally spaced intervals						P10-based scale w/o equally spaced intervals	Understands equally spaced intervals in terms of P10	
7	Epistemo-logical integration							Log scale viewed as "unreal"	Understands all scale types as "real" created tools

Figure 7. Typology of undergraduate students' conception of "size and scale."

because they, respectively, highlight the critical feature that separates fragmented conceptions from continuous ones, distinguishes conceptions based on absolute differences from proportional ones, and discriminates logarithmic conceptions from those that only include superficial log features. Essentially, these three aspects of variation differentiate the main conception categories, and thus are referred to as *between-category aspects*. In contrast, the other four aspects of variations, referred to as *within-category aspects*, highlight the subtle differences within each main category. Specifically, "integration of numbers" separates 4a and 4b conception by pointing out that 4b conception is unaware of the importance of integrating numerical measurements with scale construction; "log scale awareness" differentiates 3a and 3b conception by showing that 3b conception demonstrates no awareness of log-based system; "powers of 10" suggests that 2a conceptions are slightly more sophisticated than 2b conceptions in their awareness that proportional differences is based on a common factor of 10; and "epistemological integration" distinguishes those views which hold both logarithmic and linear scale as real, representational tools, as opposed to the belief that only the linear scale is reflective of the reality. We believe that this between versus within-category distinction is worth making because, in contrast to the within-category aspects that may be easily addressed by instruction, the between-category aspects correspond to the more conceptually difficult steps that students need to go through in order to grasp the most sophisticated conception of "size and scale" (category 1a). For example, awareness of the aspect of variation "proportion" induces a fundamental change in how students conceive of size differences—how many times as opposed to how many meters (or any other units) bigger.

As mentioned earlier, the hierarchical structure of the typology does not necessarily imply that the less sophisticated conceptions are misconceptions. They are meaningful conceptions in themselves. In other contexts, the less sophisticated conceptions could be more appropriate (e.g., linear conceptions are appropriate when students are asked to arrange items very close in size such as pen, text book, and yard stick).

Indeed, it is our belief that experts differ from novices not only in terms of the sophistication of the understanding but also in terms of the knowledge about when and which conception of a phenomenon is appropriate to apply in a particular context. Our goal of constructing the typology is to describe (as opposed to judge) the realm of conceptions students hold about "size and scale," which hopefully can serve as guidance for students to self-diagnose their own understanding, and for instructors to identify and then address the strengths and weaknesses in their students' conceptions.

Discussion

While the mounting interest in nanoscience is placing an increasing demand for schools and universities to teach relevant topics (Stevens et al., 2009; Wansom et al., 2009), little is known about how students understand one of its fundamental concepts—"size and scale." Focusing on the undergraduate population, we developed a typology that identified four categories (eight sub-categories) of conceptions, and seven aspects of variation that characterized the range of student conceptions.

Critical Components of Logarithmic Conception

Logarithmic scale is an effective tool to aid the transition from nanoscale to macroscale, which makes the logarithmic conception (category 1a and 1b) a critical level of understanding for student to have. The aspects of variation in our typology suggest that the development of this level of conception is not an "all or none" process; rather, there are several critical elements (identified by the between-category aspects of variations) that students need to integrate before fully understanding the logarithmic scale. Specifically, they need to understand that the log scale connects very big and very small objects on a continuous scale (the "continuum" aspect of variation), to conceptualize size differences (particularly large ones) in terms of proportions (the "proportion" aspect of variation), to transform proportional differences into appropriate representation on a logarithmic scale (the "equally spaced interval" aspect of variation). In other words, students need to make three key "mental switches" in their conceptual system—from the fragmented view to the continuous view of different "worlds," from the use of subtraction to division in calculating objects' size differences, and from the interpretation of each unit length on the scale as representing single unit of absolute difference (e.g., 1 m) to representing the difference of one power in terms of "powers of 10."

The importance of the "mental switch" regarding the continuous nature of scale echoes the observation by Tretter and colleagues (2006) that experts were able to view and relate different scales (e.g., nanoscale, microscale, macroscale) along a continuum, and move directly from one "world" to another depending on the context. This contrasts with the fragmented conception held by some of our participants, suggesting that understanding the continuous nature of scale is a preliminary step toward the logarithmic conception. The significance of thinking in terms of proportions was also suggested by Delgado's work (2009, 2010). After examining students ranging from middle school to undergraduate level, Delgado (2009) constructed a six-level learning progression for "size and scale." The use of proportional reasoning to connect relative scale or absolute size was one of the critical skills that characterized the more sophisticated levels of understanding. In a related study, Delgado (2010) asked undergraduate students to construct graphic representation of scales, and found that the ability to "partition an interval," which required thinking in proportional terms, to be difficult for students to grasp. Similarly, the difficulty of grasping the idea of equally spaced intervals on a logarithmic scale was also reported in a study of college students who placed historical events on a "powers of ten" number line (Confrey, 1991). The results revealed that students often had a superficial understanding of the logarithmic scale without truly understanding the meaning of equally spaced proportional intervals. The tension between additive (subtractive) and multiplicative (divisive) thinking led to a "hybrid" conception between the linear and the logarithmic scale, which is similar to the examples in the proportional conception category reported here.

Two Underlying Conceptual Dimensions

The distinctions between the observed conception categories could also be described in terms of two underlying conceptual dimensions (Figure 8). The first dimension—object versus system-based—is

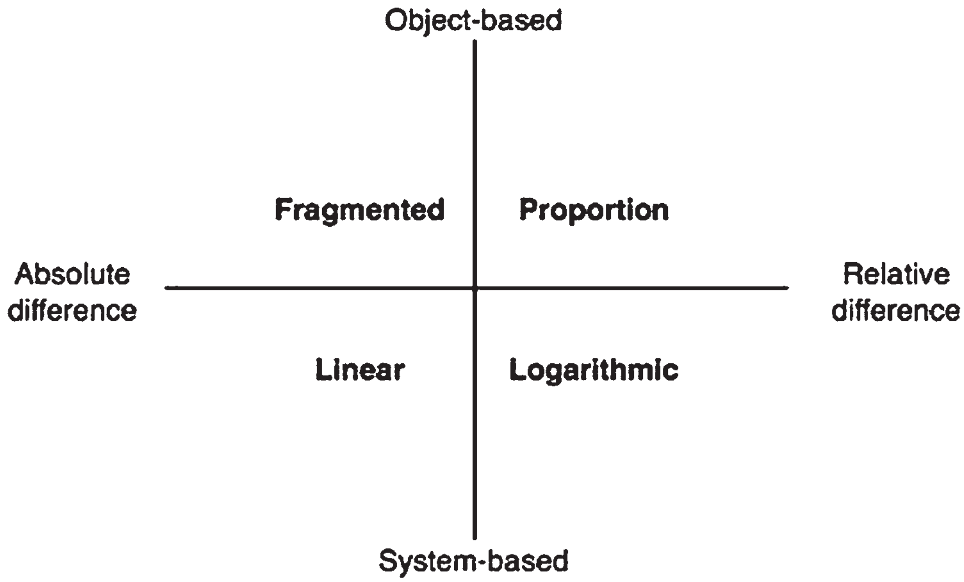

Figure 8. Conceptual dimensions underlying conceptions of ''size and scale.''

concerned with whether the size comparisons are focused on objects themselves or are placed within the framework of certain scale systems. The second dimension—absolute versus relative difference—highlights whether the size differences are conceptualized in absolute, subtractive terms or in relative, proportional terms.

Collectively, these two dimensions point out distinctions that are consistent with the between-category aspects of variation. Specifically, the fragmented and the linear conception category differ on the object versus system-based dimension, with the former focused on grouping objects of similar sizes and the latter organizing objects in one unifying system. This dichotomy highlights the essence of the ''continuum'' aspect of variation, namely whether local size differences are of primary concern (which leads to fragmented ''worlds'') or size differences across the spectrum are considered (which leads to a continuous view). Similarly, the difference between the proportional and the logarithmic conception described by the ''Equally spaced intervals'' can also be attributed to whether the construction of the scale was based on objects or scale system(s). Only in the system-based thinking, uniform intervals on the scale are necessary. The consistency between the absolute versus relative difference dimension and the ''proportion'' aspect of variation is self-explanatory, both concerned with the nature of comparisons used to gauge objects' size differences.

We view the convergence between these two underlying dimensions and the three between-category aspects of variation as an indication of the robustness of the empirically derived between-category aspects. Where students stand on these dimensions, or whether students are aware of these aspects of variation plays a determinant role in their understanding and use of the four conception categories described in our typology.

Strong Influence of Visual Experience

The observed cases of fragmented and linear conceptions in our results also suggested the strong influence of visual experience on students' understanding of ''size and scale.'' It seems that some of our participants grouped objects based on their level of visibility, and their lack of experience of ''in-between'' objects led to the belief that ''landmark objects'' (human being a common choice) separated the size continuum into disconnected ''worlds'' based on visibility. For those who held the linear view, it was clear that a ruler was visualized in their minds when comparing size differences. The fact that the placement of objects on a logarithmic scale does not match the arrangement along a ruler could also have contributed to the belief that the logarithmic scale is an artificial representational tool and does not reflect the reality.

Our students are not the only ones who relied heavily on visual experience. Even experienced professionals (Jones & Taylor, 2009) acknowledged the difficulty of conceptualizing very small or very large size scales because they cannot be easily seen. Interestingly, a study of visual impaired students (Jones, Taylor, & Broadwell, 2009) demonstrated that while their scale accuracy also decreased the further objects were removed from the human range, they were more accurate than students with normal sight in making size estimations in the small and large ends of the scale.

Instructional Implications

The considerable amount of variation revealed in our data suggested that current instruction is not effective in helping students develop a sophisticated understanding of "size and scale." One implication of our typology is that instructional emphasis should be placed on the aforementioned between-category aspects of variation.

Given students' strong reliance on visual experience, one way to help students comprehend the continuous nature of scale could be to provide students with opportunities of visualizing objects of dramatic size differences in connection to each other. In fact, the success of this approach has been documented for the use of the film *Powers of Ten* (Jones et al., 2007), which shows how things look at different size scales by zooming in and out of the human range. Stevens and colleagues (Stevens et al., 2007) also demonstrated that even limited amount of visualization exposure in a 2-week science camp was able to generate significant learning gains on size characterization. Visualization could also help students better grasp how objects differ in relation to each other, and thus begin to see the advantages of comparing them in proportional (instead of absolute) terms.

Though yet to be confirmed, our typology suggests that proportional reasoning marks a critical transition point in students' understanding of logarithmic scale, or in other words, a potentially intermediate step between the linear and the logarithmic conception. This possible trajectory receives some support from its convergence with the additive to multiplicative reasoning progression in the mathematical education literature (e.g., Confrey, 1991; Smith & Confrey, 1994), and from preliminary data collected from students of different education levels (Park, Swarat, Light, & Drane, 2010). As such, instructional activities that emphasize the differences between absolute and proportional reasoning would be useful in advancing student conceptions of not only "size and scale," but other related areas that require a solid understanding of proportional comparison.

The proposed typology, with its guiding framework of Variation Theory, is in fact quite valuable in designing such instructional interventions. Specifically, instructional activities which provide students an opportunity to become aware of the key aspects of variation distinguishing their current conception with more advanced conception(s) can be provided to help them understand the strengths and weakness of different conceptions, realize the connections between them, and consequently establish a sophisticated understanding of "size and scale." We recently piloted an instructional intervention following this strategy (Park, Swarat, Light, & Drane, 2009). Its preliminary success indicated the usefulness of the typology as a guide for instruction design.

Future Directions

Our results raised an interesting possibility that proportional reasoning is a productive intermediate step in the development of a sophisticated scale understanding. This proposed developmental path remains to be tested and verified by future studies. Furthermore, it is unclear whether the ability to grasp proportional reasoning is associated with levels of knowledge acquisition—are there certain amount of exposure to different scales, or certain accumulation of mathematical knowledge necessary for students to make the transition from absolute to proportional thinking? These questions also need to be answered, preferably with longitudinal data.

The validity of our typology, as discussed earlier in the article, is threatened by the relative small number of participants in the interview studies, and to a certain extent, the brief nature of student responses in the survey study. Further validation of the typology needs to be conducted with a larger group of students, preferably through detailed interviews, to confirm whether all of the categories and sub-categories of conceptions reported here are representative of the different ways students understand scale.

Our typology is also derived from a relatively homogenous group of students at the college level. While we believe the findings will apply to similar student population, it is unlikely that the typology is comprehensive enough to describe the conceptions of all college students, let alone students at the K-12 level. We have begun to explore high school students' conceptions of ''size and scale,'' and anticipate expanding our research to include students from different educational background in order to further verify and expand our typology.

The assessment items employed in the study were few, and were not validated through a psychometrically rigorous process. Survey items 2 and 3, as we found out, were far from ideal in eliciting a wide range of student conceptions. Although the written justifications in addition to the multiple choice responses were quite informative, we cannot rule out potential threats to the validity of the items such as soliciting biased responses or favoring only one particular type of thinking. Generating a larger pool of items and validating them through a more rigorous round of testing would be the reasonable next step.

Lastly, given the context of our studies, we placed emphasis on how students understand the logarithmic scale in the context of comparing objects' physical sizes, which is one of the many situations where the concept of ''size and scale'' could apply. It is unclear how useful our typology would be in contexts where size does not refer to spatial features such as length, area, or volume (e.g., the Richter's scale for measuring earthquake magnitude, the decibel system for measuring sound pressure level). We are interested in carrying out our research in these related areas to further validate our typology.

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Notes

[1]It should be pointed out that the choice of using smaller numbers and letters to represent the more advanced conceptions is based on the assumption that given our data were collected with undergraduate students, it is probable that we did not observe less advanced conceptions which are likely to be held by younger students. Thus our ordering of categories (and sub-categories) allows for the possibility of extending our typology based on related research conducted with k-12 students.

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